On the Existence of Fully Informative Experiment in Optimal Menu [*]

November 15, 2023

Lemma 1. *Given information structure E, $V(E,\theta)$ is continuous with respect to θ.*

Corollary 1. *The net utility $V(E(\theta),\theta)-t(\theta)$ in a feasible menu (satisfied IC and IR conditions), is lower semi-continuous with respect to θ.*

Proof. Suppose $\bar{E}$ is not in the original menu $\mathcal{M}=\{E(\theta),t(\theta)\}$.

For every type θ, θ', denote $DS(\theta,\theta')=V(E(\theta),\theta)-t(\theta)+t(\theta')-V(E(\theta'),\theta)$ and $DA(\theta,\theta')=(V(\bar{E},\theta)-V(E(\theta'),\theta))-(V(\bar{E},\theta')-V(E(\theta'),\theta'))$.

Case 1: If there exists a buyer θ' such that for any $\theta\neq\theta'$, $DS(\theta,\theta')\geq DA(\theta,\theta')>0$ or $DA(\theta,\theta')\leq 0$ holds, then we can replace the experiment of θ' as $\bar{E}$ and charge a strictly higher price $\hat{t}(\theta')=t(\theta')+V(\bar{E},\theta')-V(E(\theta'),\theta')$. Obviously, we only need to verify the IC conditions that other types $\theta\neq\theta'$ are unwilling to imitiate θ'.

$$[V(E(\theta),\theta)-t(\theta)]-[V(\bar{E},\theta)-\hat{t}(\theta')]=DS(\theta,\theta')-DA(\theta,\theta')\geq 0.$$

Case 2: If for any buyer θ', there exists a θ such that $DA(\theta,\theta')>DS(\theta,\theta')\geq 0$, denote $f(\theta)\triangleq V(E(\theta),\theta)-t(\theta)-V(\bar{E},\theta)$, then we can derive that

$$f(\theta)-f(\theta')=DS(\theta,\theta')-DA(\theta,\theta')<0.$$

,which means for any θ', there exists a $\theta\neq\theta'$ having a strictly lower value of $f(\theta)$ than that of θ'. However, by Lemma 1 and Corrollary 1, we know that $f(\theta)$ is a lower semi-continuous function over $\theta\in\Theta$. Since Θ as a simplex space is compact, by Weierstrass Theorem, there exists a θ', for any $\theta\in\Theta$, $f(\theta)\geq f(\theta')$, a contradiction. Thus for every optimal menu not containing $\bar{E}$, we feasibley adjust it to a new menu containing $\bar{E}$ which strictly increases the seller's revenue. □

References

Bergemann, Dirk, Alessandro Bonatti, and Alex Smolin, "The Design and Price of Information," *American Economic Review*, January 2018, *108* (1), 1–48.

[*] All notations are directly from Bergemann et al. (2018).